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Kaluza-Klein Type Cosmological Model of Early Universe with Variable Cosmological Term Λ: Kinematic Tests

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Abstract Kaluza-Klein type cosmological model of universe are consider with variable Λ term in the presence of perfect fluid. We obtained exact solution of the field equations by using gamma law equation of state $p=(\gamma-1)\rho$, in which parameter γ depends on the scale factor R proposed by Carvalho (Int. J. Theor. Phys. 35:2019, 1996). A unified description of early universe is presented with the assumption $\Lambda=\beta H^2$ in which *inflationary phase* is followed by *radiation dominated phase*. The various physical aspects of the cosmological models are also discussed. Analytic expressions for the look back time, proper distance, luminosity distance and angular diameter distance are derived and their meaning discussed in detail in the framework of higher dimensional space time.

Keywords Early universe · Cosmology · Cosmological constant · Inflationary phase · Radiation-dominated phase

1 Introduction

The idea that the universe we see today is only part of a higher-dimensional manifold, of which the non-visible section is too small to be resolved at currently available energies, leads in a simple and natural way to the unification of gauge and gravitational interactions (Kaluza [2], Klein [3, 4], Cho [5], Witten [6], Salam and Strathdee [7]). The Kaluza-Klein view of world geometry thus implies that the universe started out in a higher-dimensional phase with some dimensions eventually collapsing and stabilizing at a size close to the Planck length while three others continued to expand and are still doing so. This paper is motivated by the recent revival of interest in higher-dimensional field theories, in particular those of the Kaluza-Klein type (Witten [8], Cho and Freund [9]) for a list of relevant papers on the latter subject.

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A great number of exact cosmological solutions of Einsteins field equations with different equation of state and different symmetries, including or not a cosmological constant, has been found 5D (Demarat and Hanquin [10], Devidson and Vozmediano [11, 12]) and also with arbitrary number of dimension (Lorenz-Petzold [13–16]). Sahdev [17], Emelyanov et al. [18] and Chatterjee and Bhui [19], have studied physics of the universe in higher-dimensional space-time.

However, it is possible to look for effects of these extra dimensions (Chodos and Detweiler [20], Freund [21], Dereli and Tucker [22], Shafi and Wetterich [23], Randjbar et al. [24]) in the very early phases of the universe, for we evolve the Friedmann-Robertson-Walker (FRW) universe back toward the big-bang singularity, we eventually reach energies at which the extra dimensions become resolvable and, in fact, come onto the same footing as the standard dimensions.

The status of the cosmological constant problem was reviewed by Weinberg [25]. As might be expected, in the mean time, the possibility of Λ as a function of time has also been considered by Canuto et al. [26, 27]. It is worth nothing that cosmological models with varying Λ have been the subjects of numerous papers in past years. Berman [28] has studied the cosmological implications of a time dependence of the cosmological constant of the form $\Lambda \sim t^{-2}$. Some of the discussions on the cosmological constant problem and consequences on the cosmology with a time-varying cosmological constant have been discussed by Dolgov [29], Sahni and Starobinsky [30], Peebles and Ratra [31], Padmanabhan [32], Vishwakarma [33–35], Triay [36] and Kilinc [37], Bertolami [38, 39], Chen and Wu [40], Carvalho et al. [41], Berman [42, 43], Abdel-Rahman [44], Lima and Maia [45]. Al-Rawaf [46] and Overduin and Cooperstock [47] have proposed a cosmological model with a cosmological constant of the form $\Lambda \propto (\frac{\ddot{R}}{R})$. Following the same decay law, Arbab [48] has investigated cosmic acceleration with a positive cosmological constant which is equivalent to $\Lambda \propto H^2$ on dimensional ground. One of the motivations for introducing the Λ-term is to reconcile the age parameter and the density parameter of the universe with recent observational data.

Carvalho [1] and Singh [49] have studied Robertson-Walker model in general relativity by using equation of state $p = (\gamma - 1)\rho$, where the adiabatic parameter γ varies with cosmic time. His work motivate one to consider further work in some alternative theories of gravitation.

In this paper we analyzed evolution of the universe it goes from inflationary to radiation dominated phase by considering Kaluza-Klein cosmological model with variable of the form $\Lambda \propto H^2$ in the presence of perfect fluid. We also analyzed the cosmological tests pertaining to look-back time, proper distance, luminosity-distance, angular diameter distance for Kaluza-Klein type cosmological model of the universe. It has been observed that the solutions are compatible with the observations. Some physical interpretations of the cosmological solutions are also discussed.

2 Model and Field Equations

Let us consider the Kaluza-Klein type metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2\right], \tag{1}$$

where $R(t)$ is the scale factor and $k = 0, -1$ or $+1$ is the curvature parameter for flat, open and closed universe, respectively. The universe is assumed to be filled with distribution of matter represented by energy-momentum tensor of a perfect fluid

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - p g_{\mu\nu}, \tag{2}$$

where ρ is the energy density of the cosmic matter and p is its pressure and u_μ is the five-velocity vector such that $u_\mu u^\mu = 1$.

The Einstein field equations with time-dependent cosmological and gravitational constants is given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} + \Lambda(t) g_{\mu\nu}, \tag{3}$$

where $R_{\mu\nu}$ is the Ricci tensor, G is a gravitational constant and $\Lambda(t)$ being the variable cosmological constant. Using comoving coordinates

$$u_\mu = (1, 0, 0, 0, 0), \tag{4}$$

in (2) and with line element (1), Einstein's field (3) yields

$$8\pi G\rho = \frac{6\dot{R}^2}{R^2} + \frac{6k}{R^2} - \Lambda(t), \tag{5}$$

$$8\pi G p = -\frac{3\ddot{R}}{R} - \frac{3\dot{R}^2}{R^2} - \frac{3k}{R^2} + \Lambda(t). \tag{6}$$

The field equations (5)–(6) can also be written as

$$\frac{3\ddot{R}}{R} = -4\pi G\left[2p + \rho - \frac{\Lambda(t)}{8\pi G}\right], \tag{7}$$

$$\frac{6\dot{R}^2}{R^2} = 8\pi G\left[\rho + \frac{\Lambda(t)}{8\pi G}\right] - \frac{6k}{R^2}. \tag{8}$$

An over dot denotes a derivative with respective to t. The energy conservation law $T^{\mu\nu}_{;\nu} = 0$ can be written as,

$$\dot{\rho} + 4(\rho + p)H = -\frac{\dot{\Lambda}}{8\pi G}. \tag{9}$$

Equations (7) and (8) can be rewritten in terms of the Hubble parameter $H = \frac{\dot{R}}{R}$ to give, respectively

$$\dot{H} + H^2 = -\frac{4\pi}{3} G(2p + \rho) + \frac{\Lambda(t)}{6}, \tag{10}$$

$$H^2 = \frac{4\pi}{3} G\rho + \frac{\Lambda(t)}{6} - \frac{k}{R^2}. \tag{11}$$

In order to solve the field (9), (10) and (11), we assume that the pressure p and density ρ through the gamma law of equation of state

$$p = (\gamma - 1)\rho, \tag{12}$$

where γ is an adiabatic parameter varying continuously with cosmological time so that in the course of its evolution the universe goes through a transition from an inflationary phase to radiation-dominated phase. Carvalho [1] assumed the functional form of γ depends on scale factor as

$$\gamma(R) = \frac{4}{3}\frac{A(\frac{R}{R_*})^2 + (\frac{a}{2})(\frac{R}{R_*})^a}{A(\frac{R}{R_*})^2 + (\frac{R}{R_*})^a}, \tag{13}$$

where A is a constant and a is free parameter related to the power of cosmic time and lies $0 \le a < 1$. Here R_* is certain reference value such that if $R \ll R_*$, inflationary phase of the evolution of the universe is obtained ($\gamma \le \frac{2a}{3}$) and for $R \gg R_*$, we have a radiation-dominated phase ($\gamma = \frac{4}{3}$).

3 Solution of Field Equations

After imposing the condition if we put $\Lambda = \beta H^2$ in (11) (Khadekar et al. [50]), we get

$$H = C\exp\left(-\left(2 - \frac{\beta}{3}\right)\int \frac{\gamma(R)}{R}dR\right). \tag{14}$$

Substituting (13) into (14) and integrating, we have that the Hubble parameter is given by

$$H = \frac{C}{[A(\frac{R}{R_*})^2 + (\frac{R}{R_*})^a]^{\frac{2(6-\beta)}{9}}}, \tag{15}$$

where C is the integration constant.

If $H = H_*$ for $R = R_*$, we have a relation between constants A and C, is given by

$$C = H_*(1 + A)^{\frac{2(6-\beta)}{9}}. \tag{16}$$

Integrating (15) for $H = \frac{\dot{R}}{R}$, an expression for t in terms of the scale factor R, which for $a \ne 0$ is given by

$$Ct = \int \frac{[A(\frac{R}{R_*})^2 + (\frac{R}{R_*})^a]^{\frac{2(6-\beta)}{9}}}{R}dR. \tag{17}$$

By defining $q = -(\frac{R\ddot{R}}{\dot{R}^2})$, it follows from (15) that during the course of evolution the deceleration parameter is given by

$$q = \frac{(6-\beta)\gamma - 3}{3}, \tag{18}$$

which is clearly depends upon R via γ. We solve (17) for two different early phases: *inflationary* and *radiation-dominated*.

3.1 Inflationary Phase

For inflationary phase ($R \ll R_*$), the second term on right hand side of integral in (17) dominates which gives the solution for scale factor R ($a \ne 0$) as (Khadekar et al. [50]),

$$R = R_*\left[\frac{2a(6-\beta)}{9}Ct\right]^{\frac{9}{2a(6-\beta)}}, \tag{19}$$

where (19) shows that during inflation, the dimensions of the universe increase according to law $R \propto t^{\frac{9}{2a(6-\beta)}}$, which is the case of power-law inflation and for the expanding universe, we must have $0 < \beta < 6$. Equation (19) also indicates that $R = 0$ at $t = 0$, $R \to \infty$ and $\dot{R} \to 0$ as $t \to \infty$. From (19), we get the equivalent same results obtained earlier by Khadekar et al. [50].

$$H = \frac{9}{2a(6-\beta)} t^{-1}, \tag{20}$$

$$\Lambda = \frac{81\beta}{[2a(6-\beta)]^2} t^{-2}. \tag{21}$$

But the energy density is differ than Khadekar et al. [50].

$$\rho = \frac{81}{32\pi G a^2 (6-\beta)} t^{-2}. \tag{22}$$

The vacuum energy density ρ_v is given by,

$$\rho_v(t) = \frac{\Lambda}{8\pi G} = \frac{81\beta}{32\pi G a^2 (6-\beta)^2} t^{-2}. \tag{23}$$

From above set of solutions it is to be noted that $\beta \neq 6$ for physical validity. For the physical signature $\rho > 0$, we must have $0 \leq \beta < 6$. The case $\beta > 6$ is either unphysical or not compatible with time-dependent Λ. For $a > 0$ the model is singular with the energy density varying for $R \ll R_*$ as,

$$\rho = \frac{(6-\beta)C^2}{8\pi G} \left(\frac{R}{R_*}\right)^{-\frac{4a(6-\beta)}{9}}, \tag{24}$$

which is other form of energy density in terms of R. We observe that as $R \to 0$, the energy density tends to infinite. The cosmological parameter is positive and varies as the inverse square of time.

Putting the limiting value $\gamma = \frac{2a}{3}$ in (18), the asymptotic value of deceleration parameter in the limit $\frac{R}{R_*} \ll 1$, is given by,

$$q = \frac{2a(6-\beta)}{9} - 1. \tag{25}$$

This shows that the deceleration parameter is constant. By using (25), (20) can be written as,

$$H = \frac{1}{1+q} t^{-1}. \tag{26}$$

The age of the universe is given as, $t \approx mH^{-1}$ where, $m = \frac{9}{2a(6-\beta)}$. The corresponding evolution of R turns out to be $R \propto t^m$.

3.2 Radiation-Dominated Phase

For radiation-dominated phase ($R \gg R_*$), the first term on right-hand side of the integral in (17) dominates which gives the solution for scale factor

$$R = R_* \left[\frac{4(6-\beta)}{9A^{\frac{2(6-\beta)}{9}}} Ct\right]^{\frac{9}{4(6-\beta)}}. \tag{27}$$

From (27) we observe that $R \propto t^{\frac{9}{4(6-\beta)}}$, which is the case of power-law expansion and for the expanding universe, we must have $0 < \beta < 6$. From (27), we find the following solutions for Hubble parameter, cosmological constant, energy density and gravitational constant equivalent to the form Khadekar et al. [50].

$$H = \frac{9}{4(6-\beta)} t^{-1}, \tag{28}$$

$$\Lambda = \frac{81\beta}{16(6-\beta)^2} t^{-2}, \tag{29}$$

$$\rho = \frac{81}{128\pi G(6-\beta)} t^{-2}. \tag{30}$$

The vacuum energy-density ρ_v is given by,

$$\rho_v(t) = \frac{81\beta}{128\pi G(6-\beta)^2} t^{-2}. \tag{31}$$

For energy density to be positive, we must have $0 \le \beta < 6$. We also observe that $\Lambda \propto t^{-2}$ which matches with its natural dimensions. Putting the limiting value $\gamma = \frac{4}{3}$ in (18) the asymptotic value of deceleration parameter in the limit $\frac{R}{R_*} \gg 1$, is given by

$$q = \frac{4(6-\beta)}{9} - 1, \tag{32}$$

which shows that the deceleration parameter is constant.

By using (32), (28) can be written as,

$$H = \frac{1}{1+q} t^{-1}. \tag{33}$$

The age of the universe is given as $t \approx m_1 H^{-1}$ where $m_1 = \frac{9}{4(6-\beta)}$. The corresponding evolution of R turns out to be $R \propto t^{m_1}$. The other form of energy density in terms of scale factor can be written as,

$$\rho = \frac{(6-\beta)C^2}{8\pi G} \left(A \frac{R^2}{R_*^2} \right)^{-\frac{4(6-\beta)}{9}}. \tag{34}$$

One usually measures the total mass density in terms of the critical density by means of a density parameters Ω_{rad}, which is given by,

$$\Omega_{rad} = \frac{8\pi G\rho}{6H^2} = 1 - \frac{\beta}{6}. \tag{35}$$

By employing (28) and (29), this gives

$$\Omega_\Lambda = \frac{\beta}{6}. \tag{36}$$

If the universe is spatially flat, then we define Ω_{total} as

$$\Omega_{total} = \Omega_{rad} + \Omega_\Lambda. \tag{37}$$

Using (35) and (36) into (37), we get $\Omega_{total}=1$. For $\beta=0$, it is observed that $\rho\propto t^{-a}$, $R\propto t^{\frac{3}{8}}$ and $q=\frac{5}{3}$. Thus we find that the deceleration parameter varies from $q=\frac{2a(6-\beta)}{9}-1$ at $R\ll R_*$ to $q=\frac{4(6-\beta)}{9}-1$ for $R\gg R_*$. For $\beta=6(1-a)$ we have $G\propto t^{-1}$, $\rho\propto t^{-1}$ and $R\propto t^{3/8a}$. Therefore, the deceleration parameter is negative for positive cosmological constant ($\beta<\frac{15}{4}$) and for ($\beta>\frac{15}{4}$), the cosmological constant is negative.

4 Kinematics Tests

The kinematical relation distances must be confronted with the observations in order to put limits on the free parameters of the model. The formulas for $R(t)$ derived in (19) and (27) may be used to extend phenomenological analysis to arbitrary large redshifts. We now derive the kinematical relations such as look-back time, proper distance, luminosity distance and angular diameter distance for *radiation-dominated phase* of the Kaluza-Klein type model. The kinematical relations for the *inflationary phase* can be derived in similar way.

4.1 Look-Back Time

The look-back time, $\Delta t=t_0-t(z)$, is the difference between the age of the universe at present time ($z=0$) and the age of the universe when a particular light ray at redshift z was emitted. The radiation travel time for a photon emitted by a source at instant t and received at t_0 is,

$$\Delta t=t-t_0=\int_R^{R0}\frac{dR}{\dot{R}}, \tag{38}$$

where R_0 is the present scale factor of the universe. For a given redshift z, the cosmic scale factor of the universe $R(t_z)$ is related to R_0 by $1+z=\frac{R_0}{R}$, where $R=R_0$ at the present epoch. An object at $z=1$ emitted its light when the universe was half its present scale ($R=0.5R_0$). How long ago the light was emitted (look back time $\Delta t=t_0-t(z)$) depends on the dynamics of the universe. Therefore from (27), we get

$$1+z=\left(\frac{t_0}{t}\right)^{\frac{9}{4(6-\beta)}}. \tag{39}$$

The above equation gives,

$$t=t_0(1+z)^{-\frac{4(6-\beta)}{9}}. \tag{40}$$

This equation can also be expressed as

$$H_0(t_0-t)=\frac{9}{4(6-\beta)}[1-(1+z)^{-\frac{4(6-\beta)}{9}}], \tag{41}$$

which generalizes the standard FRW flat result.

For small z, (41) gives

$$H_0(t_0-t)=z-\frac{(15-4\beta)}{18}z^2+\cdots. \tag{42}$$

Using (32) in (42), we get

$$H_0(t_0-t)=z-\frac{q}{2}z^2+\cdots. \tag{43}$$

Taking the limit $z \to \infty$ in (41) the present age of the universe (the extrapolated time back to the bang) is

$$t_0 = \frac{9H_0^{-1}}{4(6-\beta)} = \frac{H_0^{-1}}{(1+q)}, \tag{44}$$

which for $\beta = 0$ reduces to the standard result.

4.2 Neoclassical Tests (Proper Distance)

A photon emitted by a source with coordinate $r = r_1$ and $t = t_1$ and received at a time t_0 by an observer located at $r = 0$ will follow a null geodesic with $(\theta, \phi) =$ constant.

The proper distance $d(z)$ between the source and the observer is given by

$$d(z) = R_0 \int_R^{R_0} \frac{dR}{R\dot{R}}. \tag{45}$$

The radial coordinate distance as function of redshift is given by

$$r_1(z) = \int_{t_1}^{t_0} \frac{dt}{R} = \frac{9H_0^{-1}R_0^{-1}}{(15-4\beta)} \left[1 - (1+z)^{\frac{4\beta-15}{9}}\right]. \tag{46}$$

Hence, proper distance as a function of redshift is given by,

$$d(z) = r_1 R_0 = \frac{9H_0^{-1}}{(15-4\beta)} \left[1 - (1+z)^{\frac{4\beta-15}{9}}\right], \quad \left(\beta \neq \frac{15}{4}\right). \tag{47}$$

For small z (47) becomes,

$$H_0 d(z) = z - \frac{2(6-\beta)}{9} z^2 + \cdots. \tag{48}$$

By using (48), we get

$$H_0 d(z) = z - \frac{(1+q)}{2} z^2 + \cdots. \tag{49}$$

From (46), it is observe that the distance $d(z)$ is maximum at $z \to \infty$ ($\beta < \frac{15}{4}$). Hence

$$d(z = \infty) = \frac{9H_0^{-1}}{(15-4\beta)}. \tag{50}$$

4.3 Luminosity Distance

The luminosity distance of a light source is defined as the ratio of the detected energy flux L, and the apparent luminosity l, i.e. $d_L^2 = \frac{L}{4\pi l}$. In the standard FRW metric (1), it takes the form

$$d_L = R_0 r_1(z)(1+z) = d_z(1+z), \tag{51}$$

where $r_1(z)$ is the radial coordinate distance of the object at light emission.

By using (46), one gets,

$$H_0 d_L = \frac{9(1+z)}{(15-4\beta)} \left[1 - (1+z)^{\frac{(4\beta-15)}{9}}\right], \quad \left(\beta \neq \frac{15}{4}\right). \tag{52}$$

For small z, after some algebra, (52) gives

$$H_0 d_L = z + \frac{1}{2}(1-q)z^2 + \cdots, \tag{53}$$

which depends on deceleration parameter.

We also note that for the same redshift the luminosity distance is larger for lower values of q. Thus, for $q = 1$, we have

$$d_L = \frac{1}{H_0} z, \tag{54}$$

and for $q = 0$, i.e. $\beta = \frac{15}{4}$ we get

$$d_L = \frac{1}{H_0}\left(z + \frac{1}{2}z^2\right). \tag{55}$$

4.4 Angular Diameter Distance

The angular diameter distance is a measure of how large objects appear to be. The angular diameter d_A of a light source of proper distance $d(z)$ is given by

$$d_A = d(z)(1+z)^{-1} = d_L(1+z)^{-2}. \tag{56}$$

By using (52), one gets

$$H_0 d_A = \frac{9}{(15-4\beta)}\left[\frac{1-(1+z)^{\frac{(4\beta-15)}{9}}}{(1+z)}\right], \quad \left(\beta \neq \frac{15}{4}\right). \tag{57}$$

Usually d_A has a minimum (or maximum) for some $z = z_c$. The angular diameter distance effect means the light is spread over a large angular area.

5 Conclusion

In the present paper we have consider Kaluza-Klein type cosmological model with varying cosmological constant and discussed the problem by applying gamma law equation of state. We have obtained a solution by assuming Λ of the form $\Lambda \propto H^2$. We have studied the behavior of scale factor R, energy density ρ and cosmological constant Λ for two different phases: *inflationary* and *radiation*. For the inflationary phase $R \propto t^{\frac{9}{2a(6-\beta)}}$ and for radiation-dominated phase we get $R \propto t^{\frac{9}{4(6-\beta)}}$. We have also studied the physical parameter H, Λ and ρ which are equivalent to results obtained earlier by Khadekar et al. [50].

We have also discussed the vacuum energy density ρ_v for both the phases in (23) and (31) respectively. The solutions represent power law and exponential expansion for $0 \leq \beta < 6$. The time dependent cosmological constant in each phase retains the natural dimension with time, i.e. $\Lambda \propto t^{-2}$. The model is expanding in each case for $0 \leq \beta < 6$. The case $\beta \geq 6$ (when $\rho \leq 0$) are either unphysical or not compatible with $\Lambda = \Lambda(t)$.

It is also shown that if universe is spatially flat, the total density parameter $\Omega_{total} = \Omega_{rad} + \Omega_\Lambda$ gives unity. This implies that the cosmological constant supplies the "missing matter" requires to make $\Omega_{total} = 1$ as suggested by the *inflationary* models, though on the basis of little observational evidence. The age of the universe in both the phases given as $t \sim mH^{-1}$, where the constant m is different in both the phases. We have also discussed the well-known astrophysical phenomena, namely the look-back time, proper distance, luminosity distance and angular diameter distance for radiation-dominated phase. It has been observed that such models are compatible with the results of recent observations and the cosmological constant Λ gradually reduces as the universe expands. It is also interesting that all the theoretical and observational results are recovered with $\beta = 0$. Thus, a unified description of early universe has been presented in which an inflationary phase is followed by radiation-dominated phase in the context of Kaluza-Klein theory of gravitation.

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